LITERATURE CITED

1. V. A. Chuzhinov, B. I. Nikolaev, N. I. Laguntsov, and G. A. Sulaberidze, "Optimization of the working regime of mass-diffusion cascade installations," At. Énerg., 38, No. 6, 363-366 (1975).
2. G. D. Rabinovich, "Optimization of the technical-economic indicators of thermal-diffusion installations for the enrichment of isotopes in the liquid phase," Inzh. Fiz. Zh., 37, No. 5, 913-917 (1979).
3. G. Dickel, "Transformationtheorie der Trennkaskaden," Nukleonik, 5, No. 8, 311-321 (1963).
4. B. Sh. Dzhandzhgava, V. A. Kaminskii, G. A. Sulaberidze, et al., "Simulation of mass-exchange processes in a mass-diffusion separation element," Inzh. Fiz. Zh., 39, No. 5, 887-892 (1980).
5. R. Ya. Kucherov and V. P. Minenko, "Theory of cascades for separating multicomponent isotope mixtures," At. Énerg., 19, No. 4, 360-367 (1965).
6. V. A. Kaminskii, A. G. Sarishvili, G. A. Sulaberidze, et al., "Taking account of countercurrent in a mass-diffusion element," At. Énerg., 49, No. 1, 57-59 (1980).
7. A. G. Sarishvili, A. A. Gvindzhiliya, V. A. Kaminskii, and G. A. Terzadze, "A mass-diffusion cascade for producing isotopes of neon," At. Énerg., 43, No. 3, 210-211 (1977).
8. D. M. Himmelblau, Applied Nonlinear Programming, McGraw-Hill (1972).
9. V. D. Borisevich, N. I. Laguntsov, and V. E. Filippov, "Optimization of functions of several variables," Inf. Buyll. Khim. Prom-ti, No. 5, 82 (1978).
10. G. D. Rabinovich, R. Ya. Gurevich, and G. I. Bobrova, Thermal-Diffusion Separation of Liquid Mixtures [in Russian], Nauka Tekh., Minsk (1971), pp. 23-178.
11. G. D. Rabinovich and R. Ya. Gurevich, "On the question of the optimal size of the separating gap in liquid thermal-diffusion columns," Inzh. Fiz. Zh., 22, No. 5, 825-828 (1972).

THERMAL CONTACT RESISTANCE IN THERMOPHYSICAL MEASUREMENTS

V. V. Kurenin

UDC 536.2

The author describes the basis for an experimental correction to the thermal contact resistance (TCR) in investigation of thermal conductivity and thermal diffusivity of solids, using specimens in the form of plates. Relations are obtained for thermal deformation of specimens and the contact pressure to compensate for thermal deflection.

The use of a permanent assembly of thermocouples in the contact plates of a thermal measurement cell simplifies the operation of the thermophysical instrument, increases its reliability, and by means of calibration allows error due to parasitic thermocouple emf's to be eliminated. However, there then arises an additional error due to thermal contact resistances (TCR) of the specimen with the thermal measurement cell. In most cases, in investigating the thermal conductivity and diffusivity of materials with $\lambda \geq 0.5$ W/(m·°K) one must introduce a correction for the TCR, determined experimentally on a specimen of known thermal conductivity, or on a metallic specimen whose thermal conductivity is considerably less than the resistance of the test specimens. For a number of practical considerations one usually prefers the second method, which is based on the hypothesis that the TCR depends only on the cleanliness of preparation of the surfaces in contact, the contact pressure, and the type of lubrication, and is independent of the thermal conductivity of the materials in contact.

It is known that heat is transmitted through a contact zone due to conduction by two methods: through the place of immediate (actual) contact of the bodies, and through the medium filling the space between rough protuberances. The conductances of the medium α_m and of the actual contact α_M are in parallel, and therefore the specific conductivity of the contact can be expressed as the sum of the specific conductivities [1]

Leningrad Institute of Technology of the Refrigeration Industry. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 42, No. 4, pp. 615-622, April, 1982. Original article submitted July 6, 1981.

0022-0841/82/4204-0425$07.50

$$\alpha_c = \alpha_{th} + \alpha_M = \frac{\lambda_m Y}{2(h_{av1} + h_{av2})} + 8 \cdot 10^3 \bar{\lambda}_M \left(\frac{p}{3\sigma_B} k \right)^{0.86}, \quad (1)$$

$$\bar{\lambda}_M = \frac{2\lambda_{M1}\lambda_{M2}}{\lambda_{M1} + \lambda_{M2}}, \quad (2)$$

where Y is the relative gap; for polished surfaces under normal conditions Y = 3.33; and

$$k = \frac{15 \cdot 10^{-6}}{h_{av1} + h_{av2}} \quad \text{for } h_{av1} + h_{av2} \leqslant 10 \ \mu\text{m}.$$

As can be seen from Eqs. (1) and (2), the first component of the conductivity is determined only by the properties of the filling medium and the cleanliness of preparation of the surfaces, while the second depends only on the conductivities of the contact materials. The contact plates of thermal measurement cells are often made of aluminum alloy, and therefore in the calibration test it is in contact with copper, and in the specimen tests, with a specimen of plastic, for example. The average thermal conductivities λ_M for the aluminum–copper and the aluminum–plastic pairs differ by a factor of 600, varying from 233 to 0.4 W/(m · °K). Hence it follows that one should use a copper specimen to determine the TCR correction only under specific conditions when the medium conductivity α_M will predominate in the contact conduction. Calculations using Eqs. (1) and (2) for typical conditions achieved in thermophysical instruments are shown in Table 1. It can be seen that optimal conditions for specimen contact are obtained with a high degree of cleanliness in the preparation of the surfaces ∇^6–∇^8 and with relatively low contact pressures $p = (1-2) \cdot 10^5$ N/m^2, and with required use of liquid lubricant. If that is so, no additional correction is needed, even in the preparation of the surfaces $\nabla 6$ due to the difference in thermal conductivities of the contact materials in the calibration and actual tests. The residual error $\Delta P_c = 9 \cdot 10^{-6}$ m^2 · K/W is insignificant for specimen thermal resistances of $P_{sp} \geq 20 \cdot 10^{-4}$ m^2 · K/W.

The data of Table 1 can be used in designing thermophysical instruments and specifying requirements for specimen preparation.

The experimental investigations show that the TCR is very unstable at low pressures $p < 50 \cdot 10^5$ N/m^2. This is associated with possible contamination of the surface in preparing the test, the presence of microdefects on the contact surfaces, and with an uneven lubricant layer. Therefore, it is especially important to investigate TCR experimentally under typical measurement conditions.

At temperatures from −150 to 400°C for dense materials, type PFMS-4 silicone oil is used successfully, for higher temperatures of TCR one can reduce the gases – argon and helium. In investigating metallic materials at medium temperatures it is appropriate to use liquid metal lubricants of eutectic In–Ga type. However, the latter requires careful handling, and cannot be recommended for wide use. In investigation of porous materials one cannot use liquid lubricants, since these, entering the material, can markedly alter its properties. Some improvement in the contact in such cases can be achieved by using dry lubricants based on finely disperse powders of good conductor material, e.g., graphite, aluminum, or silver.

TABLE 1. Calculation of TCR for Various Pairs of Materials

Type of lubricant	Contact materials	$\bar{\lambda}_M$, W/(m · °K)	Surface cleanliness	α_m W/(m² · °K)	α_M W/(m² · °K)	$P_c \cdot 10^4$, m² · °K/W	Residual error $\Delta P_c \cdot 10^4$ ∇^6	∇^7	∇^8
Air λ_m=0.03 W/(m · °K)	Aluminum–Copper λ_{m1} = 170 W/(m · °K) λ_{m2} = 370 W/(m · °K)	233	$\nabla 6$	3045	2083	1,95			
			$\nabla 7$	5314	2630	1,25			
			$\nabla 8$	10406	4672	0,66	1,35	0,65	0,3
	Aluminum–Plastic λ_{m1} = 170 W/(m · °K) λ_{m2} = 0.2 W/(m · °K)	0,4	$\nabla 6$	3045	3,6	3,3			
			$\nabla 7$	5314	4,5	1,9			
			$\nabla 8$	10406	8	0,96			
PFMS-4 oil λ_m=0.15 W/(m · °K)	Aluminum–Copper λ_{m1} = 170 W/(m · °K) λ_{m2} = 370 W/(m · °K)	233	$\nabla 6$	15228	2083	0,57			
			$\nabla 7$	26569	2630	0,34			
			$\nabla 8$	52030	4672	0,18	0,09	0,03	0,01
	Aluminum–Plastic λ_{m1} = 170 W/(m · °K) λ_{m2} = 0.2 W/(m · °K)	0,4	$\nabla 6$	15228	3,6	0,66			
			$\nabla 7$	26569	4,5	0,37			
			$\nabla 8$	52030	8	0,19			

Note: The calculations were made for values of the parameters: $P = 2 \cdot 10^5$ N/m^2; $\sigma_B = 2200 \cdot 10^5$ N/m^2; Y = 3.33; ∇^6 (h_{av} = 8.2 μm); ∇^7 (h_{av} = 4.7 μm); ∇^8 (h_{av} = 2.4 μm)

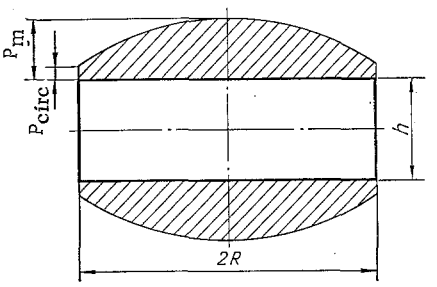

Fig. 1. Distribution of TCR over the specimen surface.

To confirm the possible use of Eqs. (1) and (2) to calculate TCR for low contact pressures we investigated TCR on the DK-*ac*λ-400 instrument [2] in the temperature range −100 to 400°C with air and type PFMS-4 oil filling the contact zone. A measurement was made on one, two, and three copper plates with identical cleanliness of preparation, which corresponded to two, three, and four contact layers. With each of the above numbers of specimens we made three tests over the entire working temperature range, each time with a new specimen. In this way we eliminated the influence of parasitic thermocouple emf's, singled out the TCR of the two copper plates, and estimated the instability of the TCR typical of the working conditions. The tests were made on specimens of diameter 15 mm with preparation cleanliness ∇7 at a pressure of $2 \cdot 10^5$ N/m^2. The TCR remained practically unchanged over the whole temperature range and turned out to be equal to that for dry contact: $(1 \pm 0.7) \cdot 10^{-4}$ m^2·°K/W; and $(0.4 \pm 0.25) \cdot 10^{-4}$ m^2·°K/W for wetted PFMS-4 oil. Calculation using Eqs. (1) and (2) for identical contact conditions gave $1.05 \cdot 10^{-4}$ and $0.32 \cdot 10^{-4}$ m^2·K/W, respectively, indicating the good agreement with the experimental data. However, it can be seen that the TCR instability is quite large. In introducing corrections to TCR in the calculation formulas for the thermal conductivity, the noneliminated part of the error, allowing for the two contact layers of the specimen, for dry contact (dc) will be $\Delta P_{dc} = 1 \cdot 10^{-4}$ m^2·°K/W and $\Delta P_{oc} = 0.35 \cdot 10^{-4}$ m^2·°K/W for wetted oil.

In addition to microroughness elements on the contact surfaces, the TCR is also influenced by the presence of the micrononuniformities, various types of deviations of the surface shape from plane-parallel.

In general, micrononuniformity sources are errors in fabrication, nonuniform wear, and mechanical and thermal deformations of the contact plates of the thermal measurement cell and specimen. Errors due to manufacture and wear are easily controlled by very simple means, for example with straight-edge templates, or by taking impressions on a flat surface. Mechanical and thermal deformations usually arise during the test and it is considerably more difficult to detect them. For design and operation of thermophysical instruments it is important to establish the allowable deviations in the shape of the contact surfaces of the thermal measurement cells. The presence of concavity or convexity on the contact plate of the thermal measurement cell introduces a systematic error into the measurement, and it is therefore desirable to investigate the possibility of allowing for this by the same methods as were used to eliminate TCR due to microroughnesses. Our interest in practice is to consider the influence of concavity and convexity of a maximum value of 0.1 to 0.2 mm.

The presence of a gap of some shape w(r) between the surface of the specimen and the isothermal contact plate is equivalent to introducing an additional TCR with the same law for distribution over the surface

$$P_c(r) = w(r)/\lambda_m .$$

We consider a specimen in the form of a planar circular plate, over the edge of which TCR is distributed in parabolic form (Fig. 1)

$$P(r) = \frac{P_{circ} - P_m}{R^2} r^2 + P_m .$$

At the edge of the specimen the TCR is determined by the microroughness, and at the center it is determined by the maximum bending. We shall make later calculations for $P_{circ} = 0.4 \cdot 10^{-4}$ m^2·°K/W and $P_m = 13 \cdot 10^{-4}$ m^2·°K/W, which corresponds to a maximum bending of 0.2 mm and to filling of the contact layer with PFMS-4 oil with $\lambda_m = 0.15$ W/(m·°K). Because of the symmetry we shall consider only one half of the specimen with a single contact layer. The total thermal resistance of this system can be obtained approximately by subdividing it with adiabats (P_Σ^a) and isotherms (P_Σ^i). Here the true value of the resistance P_Σ must lie between the values P_Σ^a and P_Σ^i.

Analysis shows that for subdivision by adiabats the thermal conduction of the system can be calculated by the formula

$$\Lambda_{\Sigma}^{a}=2\pi\int_{0}^{R}\frac{rdr}{\left(\frac{P_{circ}-P_{m}}{R^{2}}r^{2}+P_{m}+P_{sp}\right)}=\frac{\pi R^{2}}{(P_{circ}-P_{m})}\ln\left(\frac{P_{circ}+P_{sp}}{P_{m}+P_{sp}}\right),$$

and the specific thermal resistance is given by the expression

$$P_{\Sigma}^{a}=\frac{\pi R^{2}}{\Lambda_{\Sigma}^{a}}=\left[\frac{1}{(P_{circ}-P_{m})}\ln\left(\frac{P_{circ}+P_{sp}}{P_{m}+P_{sp}}\right)\right]^{-1}. \tag{3}$$

For subdivision of the system by isotherms, correspondingly, we obtain

$$\Lambda_{c}^{i}=2\pi\int_{0}^{R}\frac{rdr}{\left(\frac{P_{circ}-P_{m}}{R^{2}}r^{2}+P_{m}\right)}=\frac{\pi R^{2}}{P_{circ}-P_{m}}\ln\left(\frac{P_{circ}}{P_{m}}\right)$$

and

$$P_{\Sigma}^{i}=P_{sp}^{\prime}+\frac{\pi R^{2}}{\Lambda_{c}^{i}}=P_{sp}^{\prime}+\left[\frac{1}{P_{circ}-P_{c}}\ln\left(\frac{P_{c}}{P_{m}}\right)\right]^{-1}. \tag{4}$$

It can be seen from Eqs. (3) and (4) that the specific values of TCR do not depend on the specimen radius. In addition, in the calibration test with a copper specimen, when $P'_{sp} \ll P_{circ}$, the results of the calculation with the two formulas give identical results:

$$P_{M}^{a}=P_{M}^{i}=P_{M}.$$

From Eqs. (3) and (4) we can determine the TCR due to macrononuniformities

$$P_{c}^{a}=P_{\Sigma}^{a}-P_{sp}^{\prime}. \tag{5}$$

$$P_{c}^{i}=P_{\Sigma}^{c}-P_{sp}^{\prime}. \tag{6}$$

The calculation of the TCR of the macrononuniformities from the calibration test with a copper specimen, and also with the different subdivision systems, gives the following results: $P_M = 3.6 \cdot 10^{-4}$ m^2·°K/W; $P_c^a = 6.2 \cdot 10^{-4}$ m^2·°K/W, $P_c^i = 3.6 \cdot 10^{-4}$ m^2·°K/W.

For a specimen with an optimal thermal resistance of $P'_{sp} = 20 \cdot 10^{-4}$ m^2·°K/W, the error due to macrononuniformities lies in the range 18-31% and must be corrected for in the formula. When we introduce the correction $(0 - 2.6) \cdot 10^{-4}$ m^2·°K/W, the noncorrected error is P_M, depending on which of the two estimates Eq. (5) or Eq. (6) lies closer to the real case. A numerical calculation on a computer for specimens of 15 mm diameter and an optimal thermal resistance of $P'_{sp} = 20 \cdot 10^{-4}$ m^2·°K/W has shown that the estimate of TCR for subdivision by adiabats practically coincides with the real TCR, which is obtained by considering the two-dimensional temperature field in the specimen. Thus, for materials with thermal conductivity $\lambda_1 = 0.2$ W/(m·°K) and $\lambda_2 = 1.4$ W/(m·°K) the calculation gave a TCR of $P_{c,theory1} = 6.2 \cdot 10^{-4}$ m^2·°K/W and $P_{c,theory1} = 5.9 \cdot 10^{-4}$ m^2·°K/W, respectively.

From the results obtained we can recommend Eqs. (3) and (5) for specimens with $P'_{sp} \geq 20 \cdot 10^{-4}$ m^2·°K/W to find the limiting value of the noneliminated error due to macrononuniformities, and here the value will not be overestimated too much for $\lambda < 10$ W/(m·°K).

Thus, Eqs. (3) and (5) can be used to choose the technical limits for fabricating the contact plates and determining their allowable wear in operation. It should be noted that the formulas obtained remain valid even for a convex contact surface, when $P_{circ} > P_m$.

As was mentioned earlier, another source of macroirregularities is thermal deformation of the specimen. This arises when there is a temperature drop through the specimen thickness. Gaps appear between the contact plates of the thermal measurement cell and the specimen, and the thermal contact becomes nonuniform, which leads also to the appearance of an additional TCR at the specimen boundary. In some cases the thermal deflection of the specimen can reduce or completely eliminate the applied compression of the specimen between the contact plates. It is known from thermal stress theory [3] that the deflection of the central point of a freely supported circular plate is equal to:

TABLE 2. Calculation of Thermal Deflection and Compensating Pressure

Material	$E \cdot 10^{-8}$, N/m^2	$\beta \cdot 10^5$, 1/°K	λ, W/(m·°K)	h, mm	$P \cdot 10^4$, (m^2·°K)/W	w_T, μm	δP, %	$p \cdot 10^{-4}$, N/m^2
Amorphous polymers	15	10	0,2	1	50	84	11	0,56
Reinforced polymers	150	3	0,3	1	30	25	6	1,7
Glass	600	0,1	5	5	10	0,17	0,1	5,6
Ceramic	2000	1	7	5	7	1,7	1,6	190

$$w_T = \frac{6M_T R^2}{Eh^3}. \tag{7}$$

With a linear temperature distribution through the plate and temperature drop ϑ in it, the specific bending moment is [3]:

$$M_T = \beta E \int_{-\frac{h}{2}}^{\frac{h}{2}} t(x)\, x dx = \frac{\beta E \vartheta h^2}{12}. \tag{8}$$

By substituting Eq. (8) into Eq. (7) we can find the thermal bending of a circular plate

$$w_T = \frac{\beta \vartheta R^2}{2h}. \tag{9}$$

Assuming that the thermal bending w_T is equivalent to an additional flat layer of the same thickness with the thermal conductivity of the oil, the relative error in measuring the thermal resistance of the specimen is determined to be

$$\delta P = \frac{\Delta P}{P_{sp}} = \frac{w_T \lambda}{\lambda_m h}.$$

The compressive stress can be found by comparing the thermal deflection w_T and that of a freely supported circular plate w_p under a uniformly distributed load (pressure) p. The mechanical deflection is [4]:

$$w_p = -\frac{pR^4}{64D}\left(1 + \frac{4}{1+\mu}\right). \tag{10}$$

The cylindrical rigidity D appearing in Eq. (10) is:

$$D = \frac{Eh^3}{12(1-\mu)}. \tag{11}$$

Taking account of Eqs. (9)-(11) we find the contact pressure compensating for the thermal deformation of the specimen:

$$p = \frac{8\beta\vartheta E h^2}{3R^2(\mu^2 + 4\mu - 5)}. \tag{12}$$

Taking into account that the Poisson coefficient for most materials lies in the range 0.15–0.45, we can write Eq. (12) in a form convenient for use

$$p = (0.6 - 0.9)\beta\vartheta E\left(\frac{h}{R}\right)^2. \tag{13}$$

By way of example Table 2 shows the results of calculating the relative error δP and the compensating pressure p for materials with various physical and mechanical properties. The calculation was done for 15-mm-diameter specimens with a temperature drop of $\vartheta = 30$°K and with the contact zone filled with PFMS-4 oil with $\lambda_m = 0.15$ W/(m·°K). It can be seen from Table 2 that in the investigation of amorphous and reinforced polymers one must always provide for applied compression of the specimen, while for rigid materials - glasses, ceramics, etc., one cannot provide the stress required to avoid thermal deflection; but it is useful, in any case, to apply a pressure of $(1\text{-}2) \cdot 10^5$ N/m^2 to stabilize the contact resistance.

The basic formulas, Eqs. (3), (9), and (13), can be used also for flat specimens of more complex shape – squares, hexagons, with the proviso that for these one must substitute the effective radius found from the con-

dition that specimen areas be equal.

NOTATION

r,x, ambient coordinates; R, h, specimen radius and thickness; h_{av1}, h_{av2}, average heights of microroughnesses of the contact surfaces; w(r), deflection at the point r; w_T, w_p, deflections of the specimen center from thermal deformation and from pressure; M_T, bending moment; D, cylindrical rigidity; E, μ, Young's modulus and Poisson coefficient; σ_B, strength limit of the more plastic material; β, temperature coefficient of linear expansion; ϑ, temperature drop in the specimen; p, pressure; Y, k, relative coefficients in calculating α_m; λ, λ_m, λ_{M1}, λ_{M2}, $\overline{\lambda_M}$, thermal conductivities of the specimen, the oil, the contact materials, and the average thermal conductivity of the pair of materials in contact; α_c, α_m, α_M, specific thermal conductivity of the contact, the medium, and the actual contact; P_{sp}, thermal resistance (TR) of the specimen; ΔP_c, noneliminated part of the TCR error due to the difference in properties of the contact materials; ΔP_{dc}, ΔP_{oc}, random errors in determining the corrections to the TCR for the dry and wet contacts; P_{circ}, P_m, TR due to macrononuniformities at the edge and the center of the specimen; P_Σ, P^a_Σ, P^i_Σ, true value and approximate values of TR, obtained by subdivision by adiabats and isotherms, consisting of one half the specimen and the gap due to macrononuniformities; P^a_M, P^i_M, TR of the system consisting of one half of a copper specimen and layer with subdivision by adiabats and isotherms; P^a_c, P^i_c, $P_{c,theory}$, TR of the gap obtained by approximate subdivision by adiabats, isotherms, and numerical solution; Λ^a_Σ, Λ^i_Σ, total thermal conductivity of the system specimen – layer with subdivision by adiabats and of the layer with subdivision by isotherms; ΔP and δP, absolute and relative errors in measurement of thermal resistance of the specimen; P'_{sp}, TR of half of the specimen.

LITERATURE CITED

1. Yu. P. Shlykov, E. A. Ganin, and S. N. Tsarevskii, Contact Thermal Resistance [in Russian], Energiya, Moscow (1977).
2. E. S. Platunov, V. V. Kurepin, I. F. Shubin, et al., "The DK-acλ-400 instrument for complex thermophysical measurements," Izv. Vyssh. Uchebn. Zaved. SSSR, Priborostroenie, 15, No. 1, 131-133 (1972).
3. B. A. Boley and J. H. Weiner, Theory of Thermal Stresses, Wiley-Interscience (1960).
4. K. P. Yakovlev (ed.), Concise Physical and Technical Handbook [in Russian], Vol. 2, Fizmatgiz, Moscow (1962).